Compensation of Random and Systematic Timing Errors in Sampling Oscilloscopes

Paul D. Hale, *Senior Member*, *IEEE*, C. M. Wang, Dylan F. Williams, *Fellow*, *IEEE*, Kate A. Remley, *Member*, *IEEE*, and Joshua Wepman

Abstract--We describe a method of correcting both random and systematic timebase errors using measurements of only two quadrature sinusoids made simultaneously with a waveform of interest. We estimate the fundamental limits to our procedure due to additive noise and sampler jitter and demonstrate the procedure with some actual measurements.

Index Terms--jitter, sampling oscilloscope, timebase distortion, waveform metrology, comb generator

I. Introduction

High-speed sampling oscilloscopes suffer from systematic timebase distortion (TBD) and random jitter that cause errors in the time at which samples of a signal are acquired. We propose an alternative timebase, for use with equivalent-time sampling oscilloscopes, that greatly reduces both TBD and jitter. The new timebase relies upon simultaneous measurement of the signal of interest, and two reference sinusoids that are in quadrature and phase-locked to the signal of interest that serve to determine the actual time at which the measurement was performed [1]. The conventional timebase of the oscilloscope is used to characterize distortion in the two reference sinusoids, and to determine within which half-cycle of the auxiliary sinusoids the signal was measured. The new timebase is estimated from the sinusoids using a weighted "error-in-variables" approach that accounts for relative contributions of additive noise and timing error.

Sampling oscilloscopes that have a form of jitter correction based on quadrature sinusoidal reference signals are described elsewhere in the literature [2], and sampling oscilloscopes with similar functionality have recently become commercially available [3, 4]. Our implementation achieves the advantages of these systems, including a residual jitter of about 200 fs, correction of time records with nearly arbitrary length, and application to measurement of signals at almost any frequency. Furthermore, our method is inexpensive, since it can be implemented with an older generation of standard equipment. Our method corrects for *both* random jitter and systematic timebase distortion, and provides the user with an estimate of the residual timing error after the correction process has been applied. Also our technique is nonproprietary and is described and characterized here, for the first time, in the open archival literature.

In an oscilloscope the timing error at the *i*th sample, y_i, is the sum of the systematic TBD, h_i, and random timing jitter error τ_i. Thus the *i*th sample of the signal of interest g, as a function of time, is given by

$$y_i = g\left(T_i + h_i + \tau_i\right) + \varepsilon_i \,, \tag{1}$$

where $T_i = (i-1)T_s$ is the target time of each sample, T_s is the target time interval between samples, and ε_i is additive noise. We assume the jitter and additive noise are independent zero-mean random variables with variances σ_τ^2 and σ_ε^2.

The problem of estimating jitter and correcting for its effects has been addressed by many authors [5-9]. The typical approach is to obtain the signal variance of independent, repeated measurements and use the approximate model [10]

$$\mathrm{var}\left(y_i\right) \approx \sigma_\tau^2 \left(g'\left(t_i\right)\right)^2 + \sigma_\varepsilon^2 \tag{2}$$

to solve for σ_τ^2. Here $g'\left(t_i\right)$ is the derivative of $g(t_i)$ evaluated at $t_i = T_i + h_i$. It is usually assumed that, upon averaging, the jitter acts as a low-pass filter so that the average signal is the convolution of the signal $g(t_i)$ and the probability density function $p(\bullet)$ of the jitter:

$$\left\langle g\left(t_i\right)\right\rangle = \int g\left(t_i - \tau\right) p\left(\tau\right) d\tau \,. \tag{3}$$

The effects of jitter are then removed by deconvolution [5].

This approach has the following problems:

a) Measurements must be repeated to find the measurement mean and variance.
b) Estimates of the jitter variance from (2) are generally biased (for example, see [9]).
c) $p(\bullet)$ must be known.
d) $p(\bullet)$ must be the same over the entire measured waveform.
e) The averaging process removes some of the inherent bandwidth from the measured signal, making the deconvolution subjective[11, 12].

P. D. Hale is with the Optoelectronics Division, National Institute of Standards and Technology, Boulder, CO, 80305 USA (e-mail: hale@boulder.nist.gov).

C. M. Wang is with the Statistical Engineering Division, National Institute of Standards and Technology, Boulder, CO, 80305 USA.

D. F. Williams, K. A. Remley, and J. Wepman are with the Electromagnetic Technology Division, National Institute of Standards and Technology, Boulder, CO, 80305 USA.

Publication of the U. S. Government, not subject to U.S. copyright.

f) Deconvolution is an “ill-posed” problem [12], so that in the presence of noise there is no unique solution.

Generally, it is desirable to avoid deconvolution, particularly in cases where the jitter is large, varies over the measurement time window, or has a non-Gaussian probability density. All these situations make deconvolving the jitter from (3) problematic.

The problem of estimating TBD has also been studied by many authors [13- 20]. Recent work [17- 20] has used a nonlinear least-squares approach that fits multiple measured sinusoids with multiple phases and frequencies to a distorted sinusoid model. This approach performs well at discontinuities in the TBD and allows simultaneous estimation of the harmonic distortion, if any, in the measured sinusoids. The distorted-sinusoid model, with harmonic number n_h, is given by [18]

$$y_{ij} = \alpha_j + \sum_{k=1}^{n_h} \Big[\beta_{jk} \cos\left(2\pi k f_j t_{ij}\right) + \gamma_{jk} \sin\left(2\pi k f_j t_{ij}\right) \Big] + \varepsilon_{ij} \, , \tag{4}$$

where f_j is the fundamental frequency of the jth measured waveform y_{ij} at the ith nominal time, $t_{ij} = T_i + h_i + \tau_{ij}$. The random jitter is τ_{ij} and ε_{ij} is random additive noise. The values of α_j, β_{jk}, γ_{jk}, and h_i can be estimated, by use of a weighted least-squares approach [18]. To obtain a solution using this approach, we typically measure a set of sinusoidal waveforms at two or three different frequencies. Each set includes two sinusoids, of a given frequency, that are approximately in quadrature. Hence, each set can have up to four or six waveforms for which ε_{ij} and τ_{ij} are different for all i and j. When estimating TBD we generally average over several measurement sets to average over different realizations of ε_{ij} and τ_{ij} and reduce the uncertainty due to random jitter and additive noise. Averaging over several measured waveforms is also required in the methods described in [18] and [19].

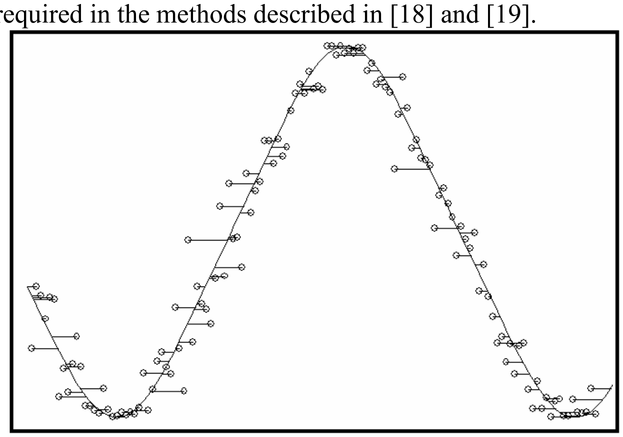

Figure 1. Circles show sampled signal using distorted and jittered oscilloscope timebase and the solid curve shows the estimated distorted sinusoid. Horizontal lines show difference between the time estimated from the curve and the nominal oscilloscope timebase.

In the present work, however, we are interested in the total timebase error, i.e. the sum of the TBD and the jitter in an *individual realization* of a measured waveform. We use all of the information in the sinusoid to find the distortion (that is, we estimate α_j, β_{jk}, and γ_{jk}) and the timebase (h_i and τ_{ij}) simultaneously so that the measured dependent variable (y_{ij}) best corresponds to the values of the distorted reference sinusoids with the new timebase. In this case, no averaging is involved.

A simple illustration is shown in Fig. 1, which plots uncorrected measurements (circles) of a reference sinusoid with an estimate of the distorted sinusoid (solid curve). Each circle represents a sample at time $t_i = T_i + h_i + \tau_i$, with each τ_i a realization of a random process. The estimated sinusoid is found by minimizing the average “distance” between the samples and the sinusoid. If we assume, for illustrative purposes, that there is no additive noise, we can estimate the total time error due to timebase distortion and jitter by drawing a horizontal line between each measurement (circles) and the distorted sinusoid. The length of each line represents the difference between the nominal (oscilloscope) time at which the measurement was taken and the time as determined by the distorted sinusoidal fit. The time that each line intersects the distorted sinusoid is the corrected time for each sample. Once the timebase error is known for each t_i, it can be applied to a simultaneously measured signal of interest *if* the timing errors of the simultaneous measurements are sufficiently correlated. In the next section we discuss how this correlation is achieved.

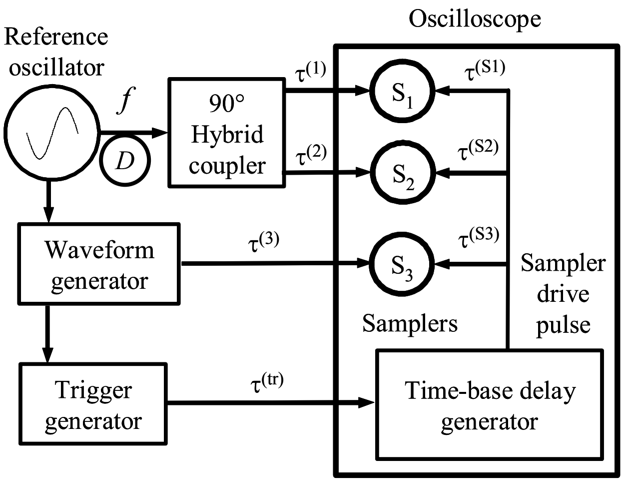

Figure 2. Schematic diagram of generic system used to measure and correct oscilloscope timebase errors. The reference generator, waveform generator, and trigger generator are synchronized. Various sources of jitter are labeled as $\tau^{(\bullet)}$.

II. SYSTEM FOR MEASURING AND CORRECTING TIMEBASE ERRORS

Fig. 2 shows a generalized schematic of the signal generator and sampling system for correcting timebase errors. The reference oscillator generates a sinusoid with frequency *f*. The waveform generator and trigger generator are synchronized to the reference oscillator. We adjust the delay *D* so that the signal propagation delay between the signal generator and samplers 1 and 2 is roughly the same as the delay between the signal generator and sampler 3. This is done to ensure minimal impact of the signal generator phase noise and maximal correlation between the reference sine waves and the waveform generator.

We take advantage of the parallel design of many equivalent-time sampling oscilloscopes. In such an oscilloscope, the sampling process proceeds as follows [21,15]: (a) the timebase is armed to trigger on a rising or falling edge at a certain level, (b) a pulse with the desired characteristics is sent into the trigger input, triggering the timebase, (c) the timebase (delay generator) waits for a predefined time delay, and then (d) the timebase generates a drive (strobe) pulse that is split and sent simultaneously to *all* the samplers in the oscilloscope mainframe. A waveform is sampled by incrementing the time delay by a nominal increment T_s and repeating the process. A result of the parallel architecture is that any jitter on the trigger pulse or the timebase delay generator is common to the sampling time of *all* the samplers in the oscilloscope mainframe.

In Fig. 2, we show the sources of jitter, measured relative to an absolute reference oscillator. They include $\tau^{(1)}$ and $\tau^{(2)}$, which are the jitter of the reference signals. We expect that these have the same statistical properties (mean of 0 and standard deviation $\sigma^{(1)} \simeq \sigma^{(2)}$), although their individual realizations for the *i*th sample might differ slightly. The value of $\tau^{(3)}$ is the jitter of the generated waveform we want to measure and has mean 0 and standard deviation $\sigma^{(3)}$. The value of $\tau^{(\mathrm{tr})}$ is the jitter of the trigger generator and timebase generator circuit and has mean 0 and standard deviation $\sigma^{(\mathrm{tr})}$. We also include a jitter τ^{Sx} $(x = 1, 2, 3)$ for the actual sampling process for each of the samplers, with mean 0 and standard deviation $\sigma^{(\mathrm{S1})} \simeq \sigma^{(\mathrm{S2})} \simeq \sigma^{(\mathrm{S3})}$.

When the samplers are simultaneously fired from the same trigger event, the different jitter components contribute to the sampled signals as follows:

$$
\begin{aligned}
S_1(t_i) &= S_1\left(T_i + h_i + \tau_i^{(1)} + \tau_i^{(\mathrm{S1})} + \tau_i^{(\mathrm{tr})}\right) \\
S_2(t_i) &= S_2\left(T_i + h_i + \tau_i^{(2)} + \tau_i^{(\mathrm{S2})} + \tau_i^{(\mathrm{tr})}\right) \\
S_3(t_i) &= S_3\left(T_i + h_i + \tau_i^{(3)} + \tau_i^{(\mathrm{S3})} + \tau_i^{(\mathrm{tr})}\right).
\end{aligned} \tag{5}
$$

We note that h_i and $\tau_i^{(\mathrm{tr})}$ are common to all the simultaneously strobed samples. Hence, if $\sigma^{(\mathrm{tr})} \gg \sigma^{(x)}$ and $\sigma^{(\mathrm{tr})} \gg \sigma^{(\mathrm{S}x)}$ (x=1, 2, 3), $\tau_i^{(\mathrm{tr})}$ is the dominant source of jitter and we can approximate τ_{ij} as $\tau_i^{(\mathrm{tr})}$. Furthermore, if we can estimate h_i and realizations of $\tau_i^{(\mathrm{tr})}$ from the known sinusoidal signals $S_1(t_i)$ and $S_2(t_i)$, we can apply our estimate to the third waveform, $S_3(t_i)$, and compensate for timing errors in its measurement.

III. ESTIMATING RANDOM JITTER

Our approach to estimating the timing errors in (4) is to apply the so called errors-in-variables [22] or orthogonal distance regression (ODR) [23] to the model in (4). In this approach, the distorted sinusoid model is fit to the data with the assumption that both "dependent" (y_i) and "independent" (t_{ij}) variables are subject to errors. Specifically, let y_{i1} and y_{i2} be the *i*th samples of nearly quadrature sinusoids measured simultaneously with the signal of interest. Denote the total timing error as $\delta_{ij} = h_i + \tau_{ij}$, $j = 1, 2$. Then δ_{i1} and δ_{i2} are the timing errors of the two sinusoid measurements. Because the samplers are driven by a common strobe pulse, as described in the previous section, we assume equal timing errors in channels 1 and 2. That is, we assume $\tau_{i1} = \tau_{i2} = \tau_i$, and hence $\delta_i = \delta_{ij} = h_i + \tau_i$ and $t_i = t_{ij} = T_i + \delta_i$. We rewrite y_{ij}, given in (4), as a function F of $\boldsymbol{\theta}_j = \left(\alpha_j, \beta_{j1} \ldots \beta_{jn_h}, \gamma_{j1} \ldots \gamma_{jn_h}\right)$ as

$$y_{ij} = F\left(T_i + \delta_i; \boldsymbol{\theta}_j\right) + \varepsilon_{ij}\ .$$

Estimates of timing errors δ_i are readily available from the ODR fit of the model using ODRPACK [23]. Although other numerical packages may also work for this application, ODRPACK has been extensively tested, shown to work well, and is freely available [24]. The ODR procedure obtains the best-fit model for this problem by minimizing the error function

$$E(\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\delta}) = \sum_{i=1}^{n} \frac{w_\varepsilon}{2}\left(\varepsilon_{i1}^2+\varepsilon_{i2}^2\right)+w_\delta \delta_i^2$$
$$=\sum_{i=1}^{n}\left\{\frac{w_\varepsilon}{2}\left(\left[F\left(T_i+\delta_i;\boldsymbol{\theta}_1\right)-y_{i1}\right]^2+\left[F\left(T_i+\delta_i;\boldsymbol{\theta}_2\right)-y_{i2}\right]^2\right)+w_\delta\delta_i^2\right\}$$

with respect to $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, and $\boldsymbol{\delta}=\left(\delta_1,\ldots,\delta_n\right)$.

The weights w_ε and w_δ are inversely proportional to the variances σ_ε^2 and σ_δ^2. That is $w_\varepsilon = c_1/\sigma_\varepsilon^2$ and $w_\delta = c_2/\sigma_\delta^2$. If the TBD does not vary greatly or an adequate TBD estimate is available as an initial guess of the timebase error δ_i, then $\sigma_\delta^2 \approx \sigma_\tau^2$ and $w_\delta = c_2/\sigma_\tau^2$. Constants c_1 and c_2 are chosen so that $\sum_{i=1}^{n} w_\varepsilon\left(\varepsilon_{i1}^2+\varepsilon_{i2}^2\right)/2$ and $\sum_{i=1}^{n} w_\delta\delta_i^2$ are about the same. Since $\sum_{i=1}^{n}\left(\varepsilon_{i1}^2+\varepsilon_{i2}^2\right)/2 \approx n\sigma_\varepsilon^2$ and $\sum_{i=1}^{n}\delta_i^2 \approx n\sigma_\tau^2$, we have $c_1=c_2$. Let $c_1=c_2=\sigma_\tau^2$, and use $w_\varepsilon=\sigma_\tau^2/\sigma_\varepsilon^2$ and $w_\delta=1$ in the estimation of timing error. We note that with these weights and the assumptions that ε_{ij} (j=1,2) and δ_i are normally distributed with mean 0 and variances σ_ε^2 and σ_τ^2, the least-squares estimators of $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, and $\boldsymbol{\delta}$ are also maximum likelihood estimators. Further discussions on the use of the weights are given in the Appendix.

IV. PRACTICAL CONSIDERATIONS

This ODR approach works well for most of the data we observe in our laboratory and requires only two nearly quadrature sinusoids. There are instances, however, where the ODR approach produces unsatisfactory results. This is the case when the waveform is very long, there are only a few samples per cycle of the sinusoid, or when the TBD is large (compared to the jitter). In such cases, we use an estimate of the TBD as an initial guess for the total timebase error to help the ODR routine converge to a solution. This initial TBD estimate requires additional measurements of quadrature sinusoids at different frequencies. These additional measurements need not be made simultaneously with the signal of interest. Criteria for frequency selection for the TBD estimate are described in detail in [17].

Additive noise on the reference sinusoids can be a source of error in any timebase error correction. From (2) we see that the frequency f of the sinusoid $g(t)$ should be chosen such that $\sigma_\tau^2\left(g'(t)\right)^2 > \sigma_\varepsilon^2$ over most of the sinusoid. That is, to achieve good discrimination between jitter and additive noise, the slew rate must be high enough so that the jitter becomes the dominant noise process for most of the sinusoid. For our sinusoid, we require $\sigma_\tau^2\left(2\pi fA\right)^2 > \sigma_\varepsilon^2$, or $2\pi f\sigma_\tau > \sigma_\varepsilon/A$, where A is the amplitude of the sine wave. We will discuss this bound further in the next section.

Table 1. Amplitude of fundamental and harmonics used in the simulation study

Fundamental frequency, GHz	Harmonic amplitude, V		
	Fundamental	Second harmonic	Third harmonic
10.0000	0.150	0.0006	0.007
9.8855	0.150	0.0006	0.007
10.2855	0.150	0.0002	0.0003

From the above discussion we conclude that we want f as large as possible. However, since we need to discriminate between half-cycles of the reference sine waves, we also require that $\sigma_\tau \ll 1/(2f)$ to make the probablility of shifting a point to the wrong quarter cycle acceptably small. Combining these limits and rearranging gives us practical bounds for selecting the frequency of the reference sinusoid: $1/2 \gg f\sigma_\tau > \sigma_\varepsilon/\left(2\pi A\right)$.

We can estimate an upper bound for the root-mean-square (RMS) residual timing error (after correction) e_Δ due to additive noise, in the limit of zero jitter, as $e_\Delta = \sigma_\varepsilon/\left(2\pi fA\right)$. For a 10 GHz sinusoid and $\left(\sigma_\varepsilon/A\right)=0.1\%$, 1%, and 5%, we obtain $e_\Delta = 0.016$ ps, 0.16 ps, and 0.8 ps, respectively.

V. SIMULATION STUDIES

We used simulation to investigate the proposed method for estimating the timing error and the fundamental limits imposed by additive noise. The criterion used in the comparisons is the amount of timing error remaining in a waveform of interest after both random and systematic timebase errors were corrected using the estimation procedure.

Recall from (1) that

$$t_i = T_i + h_i + \tau_i .$$

With estimates (denoted by ^) of the TBD, $\hat{h}_i$, and the realization of the jitter, $\hat{\tau}_i$, obtained by the estimation procedure, our estimate of t_i is then given by

$$\hat{t}_i = T_i + \hat{h}_i + \hat{\tau}_i .$$

The remaining timing errors can be characterized by the sample standard deviation, s_Δ, of

$$\Delta_i = t_i - \hat{t}_i = h_i + \tau_i - \left(\hat{h}_i + \hat{\tau}_i\right),$$

where h_i and τ_i are the actual TBD and jitter used in the simulation.

We generated sinusoids according to (4) to simulate actual measurements. The simulation parameters used here, including TBD, are closely related to those we observe in our laboratory. We used a time-measurement window (waveform epoch) of 52 ns with 53248 samples. Since the TBD would be large for this long time record, we

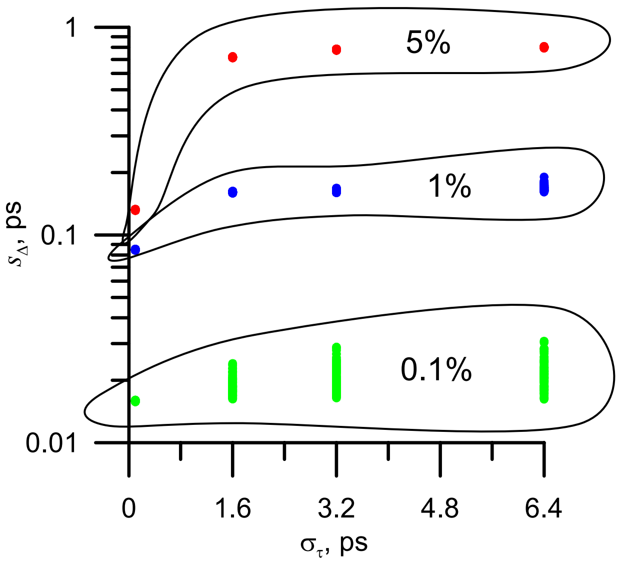

Figure 3. Sample standard deviation for all 100 simulated data sets for each of 12 different combinations of σ_ε and σ_τ. Individual symbols are not resolved in this figure.

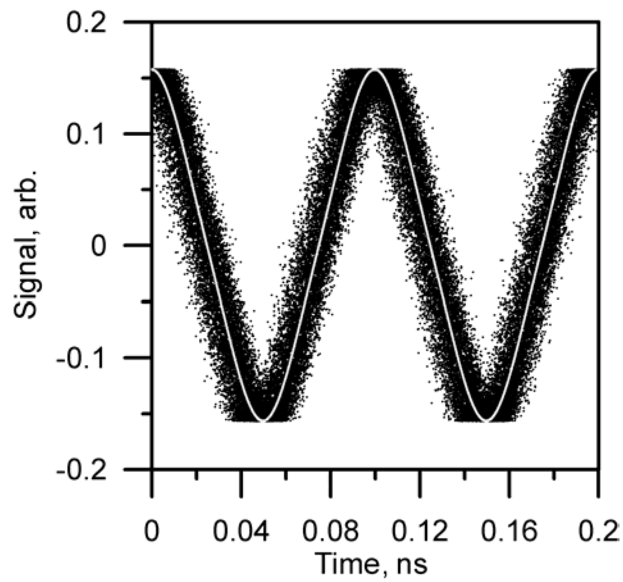

Figure 4. Plot of one of the simulated 52 ns long 10 GHz 0° sinusoids with (light dots) and without (black dots) correcting timing errors. See text for explanation.

estimated TBD and used it as an initial guess for the total time error. We generated 100 sets of 6 sinusoids, including 0° and 90° phases at three different frequencies. The signal frequencies and amplitudes are given in Table 1, along with the amplitude of the harmonics (n_h=3). In each simulation experiment, the additive noise was generated using a normal distribution with mean 0 and standard deviation σ_ε. The random jitter was generated using a normal distribution with mean 0 and standard deviation σ_τ. We also saved the nominal realization of the random jitter for the purpose of calculating Δ_i and s_Δ.

Figure 3 shows s_Δ from each of the 100 simulations of the 10 GHz 0° sinusoids for each combination of σ_ε = 0.1%, 1.0 %, and 5.0% of the fundamental amplitude and σ_τ = 0.1ps, 1.6 ps, 3.2 ps, and 6.4 ps used in the simulation experiments. Figure 3 shows that our procedure is effective for correcting the timing errors even in the presence of additive noise. Using the proper weighting for low initial jitter, allows us to achieve s_Δ that is comparable to or below the simple estimate e_Δ. In contrast, forthe case of larger initial jitter, s_Δ was approximately bounded by e_Δ.

Discussion of some particular cases in Fig. 3 is useful. For the case of σ_τ=0.1 ps and σ_ε=5.0% of the fundamental amplitude we have $f\sigma_\tau = 0.001 < \sigma_\varepsilon/(2\pi A) = 0.008$ violating our practical guidelines from Section IV. In this case, our simulations show that $s_\Delta > \sigma_\tau$. For the case of σ_τ=1.6 ps and σ_ε=5.0% of the fundamental amplitude we have $f\sigma_\tau = 0.016$, which is about two times larger than $\sigma_\varepsilon/(2\pi A) = 0.008$. In this case, our simulations show s_Δ roughly a factor of two smaller than σ_τ. Finally, in the case of very small initial additive noise, our simulations show the sample standard deviation s_Δ of the timing errors to be on the order of 0.02 ps. We will show in section VI that we can not achieve such low residual timing error because the jitter due to the samplers themselves becomes significant.

We plot one of the simulated 10 GHz 0° sinusoids with and without correcting the timing errors in Fig. 4. The long waveform (520 periods in our simulated experiments) is shown as a series of overlapping short waveforms (2 periods in this example), similar to an eye pattern. The widely scattered points are the sinusoid generated with $\sigma_\tau = 3.2$ ps and $\sigma_\varepsilon = 1\%$ of the amplitude. The overlaying (lightly shaded) points are the sinusoid after correction for timebase errors. It can be seen from Fig. 4 that after correction, the errors have been collapsed to such a small level that they can-not be resolved on this scale.

We next consider the effects of using the incorrect harmonic order in the estimation procedure. In general, the harmonic distortion that is not accounted for will have the same effect as having an inflated additive noise, with the magnitude of the effect depending on the magnitude of the distortion that is not accounted for. As an example, we simulated a signal with $\sigma_\tau = 3.2$ ps, $\sigma_\varepsilon = 1\%$ of the fundamental amplitude, but $n_h = 5$, and with the amplitudes of the actual 4th and the 5th harmonics equal to those of the 2nd and the 3rd. If we use only three harmonic terms to correct the timing errors, the mean value of s_Δ (for 100 simulations) is about 1.167 ps, a substantial increase from 0.165 ps (given in Fig. 3). However, if harmonic distortion in the 4th and the 5th is negligible we do not see a substantial increase. For example, if the amplitudes of the 4th harmonic for all three frequencies are all 0.1 mV, and the amplitudes of the 5th harmonic of the three frequencies are 0.7 mV, 0.7 mV, and 0.1 mV, then the resulting mean value of s_Δ is only 0.196 ps. It is therefore necessary to have some knowledge of the number of harmonics, n_h, which can be obtained using the method described in [18].

Weighted least-squares procedures [17-19] can also be used in place of the ODR procedure to estimate the timebase error. We used 100 simulated data sets having $\sigma_\varepsilon = 0.1\%$ of the fundamental amplitude and $\sigma_\tau = 1.6$ ps to compare the performance of the weighted least-squares and the ODR procedures. We first estimated the TBD based on the 100 measurement sets at all the three frequencies. If we used this TBD estimate as the final timebase error without further adjustments, the mean of the 100 s_Δ was found to be 1.598 ps, which, as expected, is in agreement with the initial jitter standard deviation of 1.6 ps. We then used this TBD estimate as the initial timebase error and employed the weighted least-squares [18] on each of the one-hundred 10 GHz measurements to estimate the final timebase error. The weight used for y_i is the reciprocal of $\sigma_\varepsilon^2 + \left(g'\left(t_i\right)\right)^2 \sigma_\tau^2$. The mean of the 100 s_Δ was found to be 0.84 ps, which is substantially larger than the mean of the 100 s_Δ obtained using the ODR approach (see Fig. 3).

The difference in performance between the weighted least-squares and the ODR approaches may lie in the implementation of the procedures. The algorithm implemented in the public-domain software package, ODRPACK, is an efficient and stable trust-region procedure [25]. It is more convenient to specify the model and incorporate the assumption of having common jitters between the two nearly quadrature sinusoids using the ODR approach. In addition, the package contains many error-checking facilities as well as an automatic scaling algorithm and has been extensively tested.

VI. Experimental Studies

In this section we describe experiments that verify our compensation technique. These are example measurements where timebase correction is particularly important, including cases with large jitter or long time windows where TBD can give significant errors.

A. Experimental Study 1: A single sinusoid

We tested the assumption that the trigger and timebase generator are the dominant sources of jitter ($\sigma^{(\mathrm{tr})} \gg \sigma^{(Sx)}$), which is necessary for our method to be useful, by measuring an "unknown" sinusoid (on sampler 3 of Fig. 2) that was split from the 10 GHz reference signal generator using a 3 dB splitter. The other output of the splitter was further split in a hybrid coupler to provide 0° and 90° reference signals to samplers 1 and 2 of Fig. 2. The reference signals were provided by the clock output of a digital pattern generator and the oscilloscope was triggered at 1/16 of the clock frequency using the trigger output of the pattern generator. After measuring 50 sets of these 3 sinusoids, we changed the reference frequency to the others listed in Table 1 and measured 50 sets of 0° and 90° sinusoids at those frequencies as well. Using the jitter estimation software in the oscilloscope, we estimated the jitter of the uncorrected measurement to have standard deviation of about 3.3 ps. From a separate measurement, with no input to samplers 1 and 2, we found the RMS additive noise was about 0.3% of the reference signal amplitude.

Because the sinusoid to be corrected and the reference signals are derived from the same source, we expect that the jitters $\tau_i^{(1)}$, $\tau_i^{(2)}$, and the jitter $\tau_i^{(3)}$ of the "unknown" sinusoid are highly correlated and, therefore, nearly equal. Hence, we expect this experiment to be insensitive to these parameters, with the remaining jitter being predominantly due to the jitter $\tau^{Sx}\left(x = 1, 2, 3\right)$ in the samplers.

Because of the long time record used in this experiment, we calculate the TBD as an initial guess for the ODR routine using n_h=3 and all three measured frequencies. Figure 5 shows a section of five of the 10 GHz sinusoids measured by the third sampler before (bottom) and after (top) correction for timebase errors. The uncorrected measurement has a discontinuity at 4 ns, due to timebase distortion, and the random noise is large where the slope is large, indicating significant jitter in the measurement. The corrected sinusoids have the discontinuity removed and exhibit noise that is greatly reduced and evenly distributed in time. Note that the waveforms shown in Fig. 5 have *not* been averaged.

We cannot use the procedure described in Section

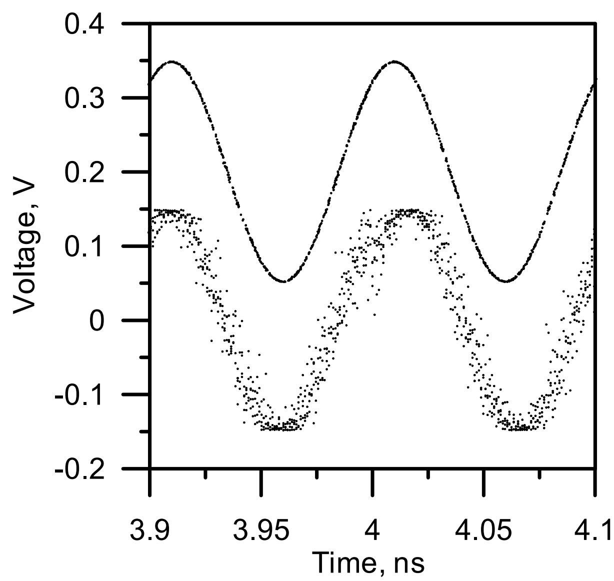

Figure 5. Portion of five sinusoids measured on sampler 3 before (bottom) and after (top) correction for time-base errors. The offset between the curves has been added for clarity.

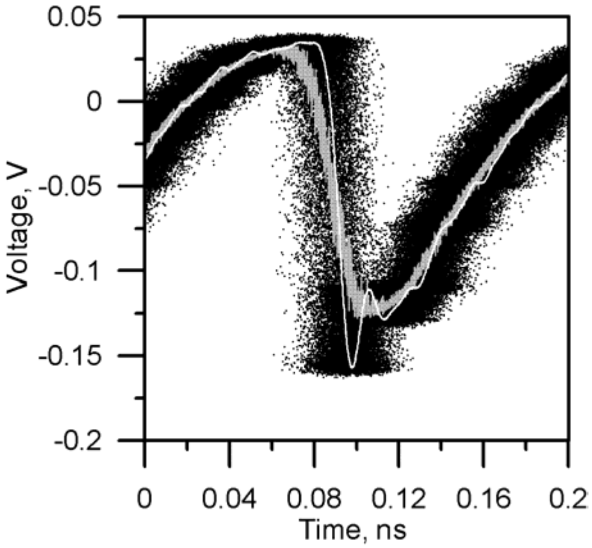

Figure 6. Comparison of raw measurement (black dots), averaged measurement (noisy gray line), and timebase-corrected and averaged measurement (smooth light line).

IV to evaluate the residual timing error because, for experimental data, both h_i and τ_i are unknown. If the waveforms of interest are known to be sinusoidal, as in this example, we can use the ODR procedure [23] to obtain an estimate of the residual timing error after correction. This is obtained from a sum of squares of the residuals of the ODR fit in the "independent" (t_{ij}) variable. The mean of the sample standard deviations of the residuals in $\hat{t}_i$ obtained from an ODR fit to 50 sinusoids measured in sampler 3 was found to be 0.2 ps. Thus, our experimental results show a jitter considerably larger than the numerical results of Fig. 3. From this we conclude that the jitter $\tau^{(Sx)}$ of the samplers is not negligible but is still much smaller than the original jitter in the measurement. We estimated the jitter of one of the samplers using the estimated numerical limit of 0.021 ps, for our initial jitter and additive noise from Fig. 3, as $\sqrt{0.2^2 - 0.021^2}\big/\sqrt{2} = 0.14$ ps, where we have divided by $\sqrt{2}$ to reflect the even distributed of the jitter between sampler 3 and the sampler that is predominantly used as the reference signal for any given sample. This gives an estimated lower bound to our timebase correction due to sampler jitter; $0.14\sqrt{2} \approx 0.2$ ps. Although the sampler jitter is not negligible, it is 23 times smaller than the initial jitter in this experiment and about a factor of 6 smaller than the lowest jitter we observe in any of our laboratory measurements. We conclude that $\sigma^{(\mathrm{tr})}$ is sufficiently large compared to $\sigma^{(Sx)}$ and therefore expect reduced timebase error by using our procedure.

B. Experimental Study 2: Fast transient with jitter

In some measurement situations, such as those requiring averaging, (3) shows that jitter will blur details of a fast transient event, such as the output of a comb generator used for calibrating various high-speed measurement equipment. In the context of this work, measurement of a fast transient allows us to use (2) to obtain an estimate of the residual jitter, after our correction, that is independent of the ODR algorithm. As stated before, jitter estimates made using (2) will have some bias, but have sufficient accuracy for the present purposes.

Table 2. Residual jitter on measured NLTL waveform

Initial rms jitter as measured on oscilloscope, ps	Residual rms jitter after correction, ($w = \sigma_\tau^2 \big/ \sigma_\varepsilon^2$), ps
1.4	0.15
3.0	0.20
6.3	0.25
8.6	0.25

To generate our fast transient, we used a 6 GHz signal generator to drive a nonlinear transmission line (NLTL). The NLTL was configured to steepen the falling edge of the generated sinusoid, giving a fast transient with a 6 GHz repetition rate. The output of the signal generator was split between a countdown trigger generator, used to trigger the oscilloscope, the NLTL, and a hybrid coupler whose outputs were used as the reference signals on samplers 1 and 2. The measured transient from the NLTL (without deconvolution of the oscillscope impulse response) has roughly a 9 ps fall time.

By changing the trigger level of the oscilloscope we can change the root mean square (rms) jitter from about 1.4 ps to more than 8.6 ps (as measured by the oscilloscope). Additive noise on the reference signals was about 0.4% of the sinusoid amplitude. Figure 6 shows 50 measurements of the waveform generated by the NLTL before averaging (black dots) and after averaging (noisy gray line) for the case where the rms jitter is 8.6 ps. The light smooth curve in Fig. 6 is the result of the following correction and averaging procedure: (a) each of the 50 waveforms was corrected for timebase errors, (b) each corrected waveform was linearly interpolated back to the original evenly spaced time grid, and (c) the resulting curves were averaged. This estimated waveform has much less noise but has ripple, ringing, and sharp features that are blurred in the corresponding average of the uncorrected measurements.

Figure 7 shows an expanded view of the waveform after applying our procedure, with three different initial values of jitter. Notice that the curves lie nearly on top of each other. Because the ringing and ripple are accurately represented in each reconstructed waveform, these features are *not* artifacts of the signal processing, as might be expected with some kinds of regularized noncausal deconvolution [11].

Closer inspection of the curves in Fig. 7 shows systematic time differences in the curves that increase with initial jitter, but are still substantially less than the initial jitter. The two lowest jitter curves typically agree to within

100 fs, while the lowest and highest jitter cases differ by as much as 1.1 ps at some times. This systematic difference between the high- and low-jitter cases may be caused by the high noise level of the high jitter case which leads to poor estimation of the harmonic content in the reference signal. It should be noted that 8.6 ps is on the order of 3 to 10 times larger than the jitter we observe in typical measurements. Further investigation of this source of error is beyond the scope of this work. The calculated fall times (10-90% of peak-to-peak transition durations) of all four cases are indistinguishable.

Because we do not have an analytic expression for the fast transient, we cannot use the ODR approach to estimate the residual timing error in its measurement after correction. To estimate the residual jitter in the transient measurement we used (2) on the corrected and linearly interpolated waveforms. Interpolation to a uniform grid allows us to estimate the variance and derivative at a given time, as is needed in (2). The results of our estimate are shown in Table 2. We observe that the results for the experiments with most similar initial jitter (3.2 ps in sinusoidal experiment, 3.0 ps in NLTL experiment) are in good agreement; both have 0.2 ps residual jitter after correction and include the same amount of error from sampler jitter. Table 2 also shows that the resulting residual jitter is only weakly dependent on the initial jitter, as expected from the simulations in Section IV, and that the algorithm can improve a high jitter measurement by as much as 34× (from 8.6 ps to 0.25 ps).

VII. Demonstration Program

Our program for post-processing acquired waveforms for timebase correction has a graphical user interface that can be used in a Microsoft Windows® [26] environment. The program, available at http://www.boulder.nist.gov/div815/HSM_Project/HSMP.htm, contains examples of how the software can be used to correct single or multiple measurements. These examples can be accessed through the program's help menu under "Getting Started". Instructions are also given on how to call the program from other programs with ActiveX® [26] capability.

VIII. Conclusion

We have shown how to simultaneously estimate the systematic and random timebase errors of measured sinusoidal reference signals. Using the parallel (simultaneous) sampling in our oscilloscope allows us to use this estimate to correct the timebase errors in a simultaneously measured waveform by roughly a factor of 10, effectively replacing the timebase of the oscilloscope with a timebase provided by the measured sinusoids. We require only that the oscilloscope timebase have enough accuracy to allow us to discriminate between consecutive cycles of the clock signal. This allows us to correct the timing errors that might be present with long waveforms or large jitter, and lowers the noise floor significantly in most measurements without averaging. In addition to the examples described in this paper, we have also demonstrated clear reduction of effects due to random jitter and timebase distortion in measurements of 10 Gbit/s data sequences that are 52 ns (53248 samples) long and multisine signals that are 500 ns (40960 samples) long.

IX. Acknowledgements

We gratefully acknowledge the loan of the NLTL by Martin Van Pelt, and help with generating the Dynamic link libraries (DLLs) for the Visual Basic interface from Abbie O'Gallagher. We also thank Tracy Clement, Jonathan Scott, Don Larson, and the journal reviewers for their detailed review and useful comments that improved

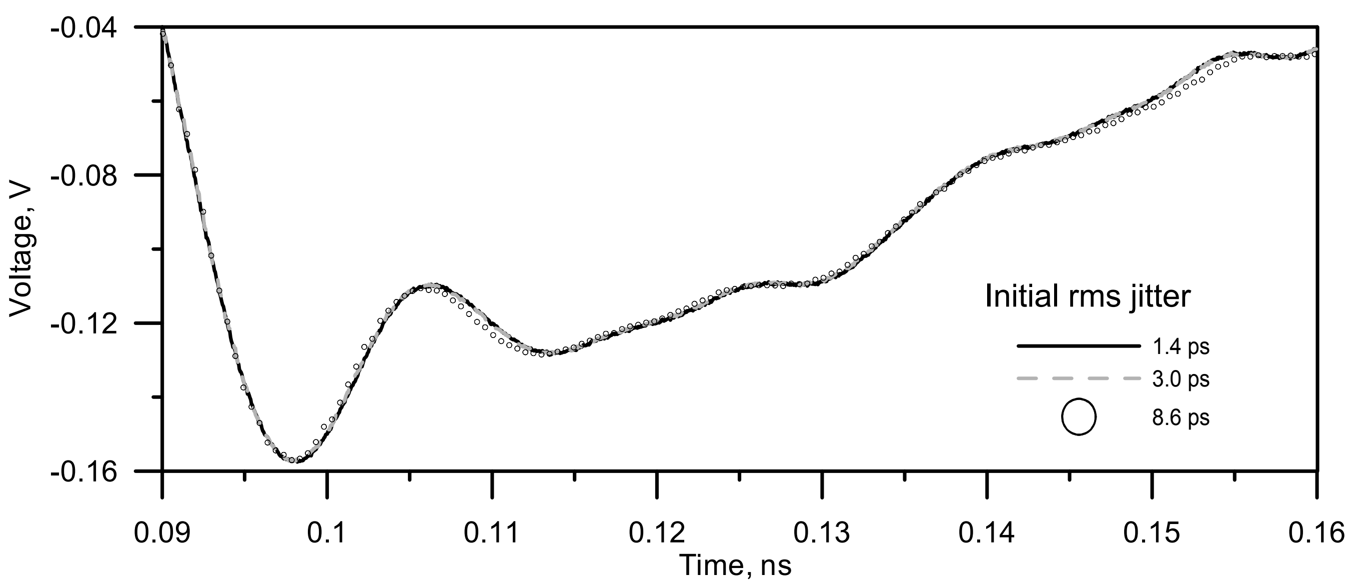

Figure 7. Comparison of some corrected and averaged measurements. Measurements with initial jitter of 1.4 ps and 3.0 ps are indistinguishable on this scale, while the measurement with initial jitter of 8.6 ps shows differences as large as 1.4 ps at some times.

our manuscript.

X. Appendix

If σ_τ^2 and σ_ε^2 are not known or can not be accurately estimated, the following procedure may be used to obtain an approximate estimate of the relative weight $\sigma_\tau^2 / \sigma_\varepsilon^2$. The procedure is based on the assumption that an adequate TBD estimate is available as an initial guess of the timebase error.

The procedure first estimates the timing errors using $w_\varepsilon = w_0 = 1\,\mathrm{ns}^2/\mathrm{V}^2$. Let S_δ and S_ε be the weighted sums of squared residuals for δ and ε , respectively, from the ODR fit. If $S_\delta \approx S_\varepsilon$, then the correct weight has been used. Otherwise, use the new weight $w_\varepsilon = w_0 S_\delta / S_\varepsilon$ in the next ODR fit. And repeat this process until $S_\delta \approx S_\varepsilon$. For example, for the case where σ_ε =0.1% of the fundamental and σ_τ =6.4 ps in the simulated experiment of section V, using w_ε =1 ns^2/V^2 in the ODR fit produces S_δ =2.124 and S_ε =0.02572 for the first set of measurements. (For illustration, we only report the results for the first set of measurements. Results for the other 99 sets of measurements are very similar.) A new weight of w_ε =(2.124)/(0.02572)=82.58 in the next ODR fit produces S_δ =2.173 and S_ε =0.09923. The next weight to use is w_ε =(82.58 ns^2/V^2) $\cdot$ (2.173)/(0.09923)=1808.388, which produces S_δ =2.174 and S_ε =2.164. The correct weight for this problem is 1820.4 ns^2/V^2 . For the case where σ_ε =5% of the fundamental and σ_τ =3.2 ps, one iteration yields a weight of 0.188 ns^2/V^2, which is close to the correct weight of 0.182 ns^2/V^2. For other combinations of σ_ε and σ_τ , it generally requires 1 or 2 iterations to obtain a “close” estimate of the correct relative weight.

[1] S. H. Pepper, “Synchronous sampling and applications to analytic signal estimation,” *56th ARFTG Conf. Dig.*, pp.170-179, Dec. 2000.
[2] R. L. Jungerman, G. Lee, O. Buccafusca, Y. Kaneko, N. Itagaki, R. Shioda, A. Harada, Y. Nihei, and G. Sucha, “1-THz Bandwidth C- and L-band optical sampling with a bit rate agile timebase,” IEEE Photon. Technol. Lett., vol. 14, pp. 1148-1150, 2002.
[3] Agilent 86107A precision timebase reference module. NIST does not endorse or guarantee this product. This product is listed here only to reference similar measurement techniques. Other products may perform as well or better than those listed here.
[4] Tektronix 82A04 phase reference module. NIST does not endorse or guarantee this product. This product is listed here only to reference similar measurement techniques. Other products may perform as well or better than those listed here.

[5] W. L. Gans, “The measurement and deconvolution of time jitter in equivalent-time waveform samplers,” *IEEE Trans. Instrum. Meas*., vol. 32, pp. 126-133, 1983.
[6] J. Verspecht, “Compensation of timing jitter-induced distortion of sampled waveforms,” *IEEE Trans. Instrum. Meas*., vol. 43, pp. 726-732, 1994.
[7] G. Vandersteen and R. Pintelton, “Maximum likelihood estimator for jitter noise models,” *IEEE Trans. Instrum. Meas*., vol. 49, pp. 1282-1284, 2000.
[8] M. G. Cox, P. M. Harris, and D. A. Humphreys, “An algorithm for the removal of noise and jitter in signals and its application to picosecond electrical measurement,” Numerical Algorithms, **5**, pp. 491-508, 1993.
[9] K. J. Coakley, C. M. Wang, P. D. Hale, and T. S. Clement, “Adaptive characterization of jitter noise in sampled high-speed signals,” *IEEE Trans. Instrum. Meas*., vol. 52, pp. 1537-1547, 2003.
[10] T. M. Souders, D. R. Flach, C. Hagwood, and G. L. Yang, “The effects of timing jitter in sampling systems,” *IEEE Trans. Instrum. Meas*., vol. 39, pp. 80-85, 1990.
[11] N. S. Nahman and M. E. Guillaume, *Deconvolution of Time Domain Waveforms in the Presence of Noise*, NBS Technical Note 1047, National Bureau of Standards, Oct. 1981.
[12] *Solutions of ill-posed problems*, A. N. Tikhonov and V. Y. Arsenin, V. H. Winston and Sons, Washington, D. C., 1977.

[13] W. R. Scott Jr., “Error corrections for an automated time-domain network analyzer,” *IEEE Trans. Instrum. Meas*., vol. 35, pp. 300-303, 1986.
[14] J. Verspecht, “Accurate spectral estimation based on measurements with a distorted-timebase digitizer,” *IEEE Trans. Instrum. Meas*., vol. 43, pp. 210-215, 1994.
[15] J. B. Retting and L. Dobos, “Picosecond time interval measurements,” *IEEE Trans. Instrum. Meas*., vol. 44, pp. 284-287, 1995.
[16] “IEEE standard for digitizing waveform recorders,” IEEE Std. 1057-1994, 1994.
[17] G. N. Stenbakken and J. P. Deyst, “Timebase nonlinearity determination using iterated sine-fit analysis,” *IEEE Trans. Instrum. Meas*., vol. 47, pp. 1056-1061, 1998.
[18] C. M. Wang, P. D. Hale, and K. J. Coakley, “Least-squares estimation of timebase distortion of sampling oscilloscopes,” *IEEE Trans. Instrum. Meas*., vol. 48, pp. 1324-1332, 1999.
[19] G. Vandersteen, Y. Rolain, and J. Schoukens, “An identification technique for data acquisition characterization in the presence of nonlinear distortions and timebase distortions,” *IEEE Trans. Instrum. Meas*., vol. 50, pp. 1355-1363, 2001.
[20] C. M. Wang, P. D. Hale, K. J. Coakley, and T. S. Clement, “Uncertainty of oscilloscope timebase distortion estimate,” *IEEE Trans. Instrum. Meas*., vol. 51, pp. 53-58, 2002.
[21] J. Verspecht, “Calibration of a measurement system for high frequency nonlinear devices,” Ph.D. Thesis, Free University of Brussels, Brussels, Belgium, Sept. 1995.
[22] W. A Fuller, *Measurement error models*, NY: John Wiley, 1987.
[23] P. T. Boggs, R., H. Byrd, J. E. Rogers, and R. B. Schnabel, “User’s reference guide for ODRPACK version 2.01 software for weighted orthogonal distance regression,” Nat. Inst. Stand. Technol. Internal Report, NISTIR 92-4834, June 1992, available at http://www.boulder.nist.gov/mcsd/Staff/JRogers/odrpack.html.
[24] ODRPACK is freely available at http://www.netlib.org/netlib/odrpack/
[25] P. T. Boggs, R. H. Byrd, and R. B. Schnabel, ``A stable and efficient algorithm for nonlinear orthogonal distance regression," SIAM, J. Sci. Statist. Comput. vol. 8, pp. 1052-1078, 1987.
[26] NIST does not endorse or guarantee this product. This product is listed here only to provide the reader with information on how to use the software. Other products may perform in this application as well or better than those listed here.